fect of the temperature factor must then be considered just as in a pure gas. It can be proposed that for the case of a nonblack wall at emissivities of $\varepsilon_w = 0.8$-0.9, a correction factor equal to $0.5(\varepsilon_w + 1)$ should be introduced on the right side of Eq. (6).

NOTATION

ρ_*, h_*, mixture density and enthalpy; r, r_0, radial coordinate and tube radius; $R = r/r_0$, dimensionless radius; η_{res}, resultant radiation flux volume density; q_r, resultant radiation flux density; α, absorption coefficient; $\overline{T}$, mean mass temperature of mixture; $T_* = \overline{T}$, if $T_w < \overline{T}$; $T_* = T_w$, if $T_w > \overline{T}$; K, particle mass flow concentration; $I_0(x)$, $K_0(x)$, $K_1(x)$, $I_1(x)$, modified Bessel functions; ε, particle cloud emission coefficient; ℓ_{ef}, effective beam length; σ, Stefan–Boltzmann constant; $Bu = \alpha r_0$, Buger number. Subscript w, wall.

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UTILIZATION OF THE K–ε TURBULENCE MODEL IN A FREE-CONVECTIVE TURBULENT BOUNDARY LAYER

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A free-convective turbulent boundary layer on a vertical isothermal surface is examined. The influence of the buoyancy force on the kinetic energy of the turbulent fluctuations is analyzed. A modification is proposed for the turbulence model that takes account of the free-convective flow singularities.

Modeling turbulence when studying free-convective boundary layers is based mainly on the analogy with forced flows [1, 2] without taking account of the influence of the lift force on the turbulent characteristics. Experimental papers [3-6] that have recently appeared and in which the structure of a turbulent free-convective flow is investigated in detail permitted substantial refinement of the turbulence model and taking account of the singularities of similar flows.

As the initial equations to describe the free-convective flow around an isothermal vertical surface, the turbulent boundary-layer equations in a Boussinesq approximation were used. Details of the problem formulation can be found in [7].

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The turbulent viscosity coefficient ν_T was determined from the formula [1]

$$\nu_T = c_\mu \exp\left[-2.5/(1 + \mathrm{Re}_T/50)\right] \frac{K^2}{\varepsilon}. \tag{1}$$

To determine the kinetic energy of turbulent fluctuations the appropriate transport equation can be used

$$u\frac{\partial K}{\partial x} + v\frac{\partial K}{\partial y} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\mathrm{Pr}_K}\right)\frac{\partial K}{\partial y}\right] - \langle u'v'\rangle \frac{\partial u}{\partial y} + g\beta\langle u'T'\rangle - \varepsilon - 2\nu\left(\frac{\partial K^{1/2}}{\partial y}\right)^2. \tag{2}$$

Here the x axis is directed along the surface and is parallel to the free-fall acceleration g.

Correlation of the velocity fluctuations $\langle u'v'\rangle$ when using the Boussinesq hypothesis is written as follows:

$$-\langle u'v'\rangle = \nu_T \frac{\partial u}{\partial y}. \tag{3}$$

The correlation $\langle u'T'\rangle$ that takes account of the lift coefficient in the magnitude of the turbulence kinetic energy was determined from the formula [8]

$$\langle u'T'\rangle = c_4 (qK)^{1/2}. \tag{4}$$

In its turn, the quantity q was found from the solution of the appropriate transport equation [1]

$$u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{\nu}{\mathrm{Pr}} + \frac{\nu_T}{\mathrm{Pr}_q}\right)\frac{\partial q}{\partial y}\right] + c_{1q}\nu_T\left(\frac{\partial T}{\partial y}\right)^2 - c_{2q}\frac{\varepsilon}{K}q - 2a\left(\frac{\partial q^{1/2}}{\partial y}\right)^2. \tag{5}$$

The rate of turbulence kinetic energy dissipation was determined from the following equation [1]

$$\begin{aligned} u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\mathrm{Pr}_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + c_1\frac{\varepsilon}{K}\nu_T\left(\frac{\partial u}{\partial y}\right)^2 - \\ - c_2\left[1 - 0.3\exp(-\mathrm{Re}_T^2)\right]\frac{\varepsilon}{K} + c_3 c_4 g\beta (qK)^{1/2}\frac{\varepsilon}{K}. \end{aligned} \tag{6}$$

The introduction of additional terms in the equations for K and q [see the last term in (2) and (5)] permitted taking account of the anisotropy of turbulence near the solid surface and the utilization thereby of the zeroth value as the boundary condition for ε. Consequently, the boundary conditions for K, q and ε have the form

$$\begin{gathered} K = 0,\ q = 0,\ \varepsilon = 0 \quad \text{for } y = 0, \\ K \to 0,\ q \to 0,\ \varepsilon \to 0 \quad \text{for } y \to \infty. \end{gathered} \tag{7}$$

The initial system of boundary-layer equations in combination with (2), (5), and (6) was approximated by finite differences of second-order accuracy. The difference system of algebraic equations obtained was solved by the method of nonmonotonic factorization with iterations. Details of the numerical method can be found in [9]. The following values of the empirical constants were used in the computation: $c_\mu = 0.09$, $c_{1q} = 2.8$, $c_{2q} = 1.7$, $c_1 = 1.44$, $c_2 = 1.92$, $c_3 = 1.44$, $c_4 = 0.5$, $\mathrm{Pr}_q = 0.9$, $\mathrm{Pr}_\varepsilon = 1.3$.

Profiles of the quantities $k = K/K_{max}$, $e = \varepsilon/\varepsilon_{max}$ (curves 1 and 3) are compared in Fig. 1 with analogous profiles obtained in [1] (curves 2 and 4). It is seen that in contrast to the results in [1], the nonmonotonic nature of the changes in k and e in the domain $0.01 \le y/\delta \le 0.2$ is observed in the results represented in this paper.

It must be noted that as the accuracy of the convergence of the iteration process diminishes, a solution is obtained with a monotonic change in the quantities k and e, i.e., exactly as in [1].

The condition

$$\left|\frac{\nu_T^{(i)} - \nu_T^{(i-1)}}{\nu_{T\,max}}\right|_j \leqslant \zeta, \tag{8}$$

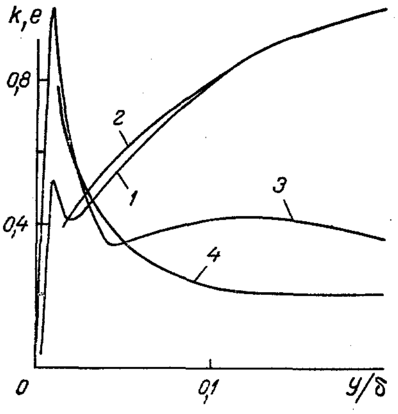

Fig. 1

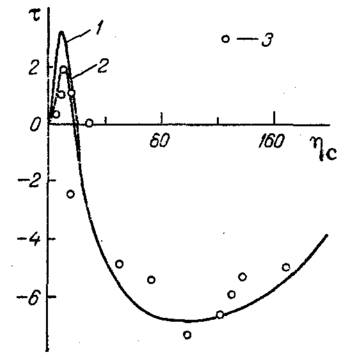

Fig. 2

Fig. 1. Dependence of k (curves 1, 2) and e (curves 3, 4) on y/δ: 1, 3) without taking account of singularities in the behavior of ⟨u'T'⟩; 2, 4) from computed data [1], Gr = $4.8 \cdot 10^{10}$.

Fig. 2. Dependence of τ on η_c: 1) without taking account of singularities in the behavior of ⟨u'T'⟩; 2) using the factor (9); 3) from experimental data [4]: Gr = $1.62 \cdot 10^{10}$.

was the final criterion for emergence from the iterations in the computations we made, where $1 < j < N$ is the number of points in a direction perpendicular to the wall, N is the quantity of computational nodes in the layer, i is the number of iterations, and ζ is a certain small quantity.

By using an additional numerical investigation it was established that the results of [1] can be duplicated only for $\zeta \geq 5 \cdot 10^{-2}$. As the accuracy of the convergence of the iterations increases, nonmonotoneity in the behavior of k and e appeared and was conserved even in the final solution. A known finite-difference method [10] was selected in [1] for the numerical solution of the system of equations. This method cannot apparently assure the necessary accuracy for the solution of similar problems.

Comparison of the dimensionless tangential friction (see Fig. 2, curve 1) and the dimensionless generation of turbulence because of the shear flow p (see Fig. 3, curve 1) with available experimental data [3, 4] showed that a substantial exaggeration of these quantities is observed in the near-wall domain.

Starting from the results obtained, we assume that the behavior of the free-convective turbulent boundary layer near a surface is anomalous. The qualitative analysis performed in [11], where the existence of a domain with negative correlation values ⟨u'T'⟩ near the wall is indicated, confirmed this assumption. Moreover, the presence of negative values of the quantity ⟨u'T'⟩ in the domain between the surface and the maximum velocity coordinates was verified experimentally in [5, 6].

Therefore, the contribution of the buoyancy force to the turbulence kinetic energy is different near to and far from the wall. In other words, taking account of the Archimedes force diminishes the turbulence kinetic energy in the domain between the surface and the maximum velocity coordinate and increases it in the external boundary layer domain.

The singularity noted influences the nature of the turbulent free-convective flow and should be taken into account in modeling turbulent processes. It is certainly impossible to take account of a similar turbulent flow singularity by using the turbulence model being applied; thus, for instance, the structure of (4) is such that it is generally impossible to obtain a negative value of ⟨u'T'⟩.

In this paper it is proposed to take into account redistribution of the energy between the fluctuating and average motion by using an additional factor in the expression (4):

$$A_{\text{т}} = \begin{cases} 1 - 1/(a_1 \text{Ri}_{\text{т}}^{b_1}), & y \leqslant y_{\text{м}}, \\ 1, & y > y_{\text{м}}. \end{cases} \tag{9}$$

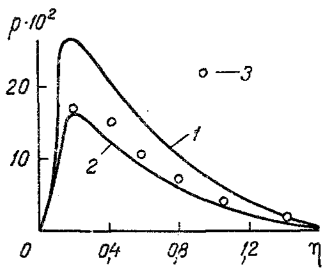

Fig. 3

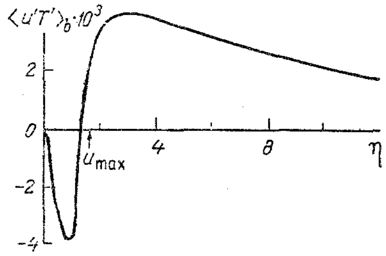

Fig. 4

Fig. 3. Dependence of p on η: 1) without taking account of singularities in the behavior of ⟨u'T'⟩; 2) using the factor (9); 3) from experimental data [3]; Gr= $5.57 \cdot 10^{10}$.

Fig. 4. Distribution of $\langle u'T' \rangle_b$ in the transverse section of a free-convective boundary layer: Gr = $1.62 \cdot 10^{11}$.

The values of the constants a_1 and b_1 were determined as a result of a numerical experiment (a_1 = 50, b_1 = 1.45).

Profiles of the quantities τ and p in Figs. 2 and 3 (curve 2), obtained with the factor (9) taken into account, are compared with experimental results [3, 4]. Application of the factor A_T results in a monotonic change in K and ε in the domain $0.01 \le y/\delta \le 0.2$ and substantially improves the agreement between the computed and the experimental data in the near-wall domain.

On the basis of the algebraic model of Roddy for stresses in a locally equilibrium approximation an analysis is given in [11] of the behavior of different correlations in a turbulent free-convective vertical boundary layer, and it is shown that the correlation ⟨u'T'⟩ is negative in the domain between the wall and the maximum velocity. However, it is also noted in [11] that the locally equilibrium approximation is too inaccurate near the wall and, therefore, the derivation of a negative value of the correlation ⟨u'T'⟩ in this domain has an incomplete foundation.

The complete algebraic model of Roddy was used in this paper in the presence of expulsion forces to determine the stresses [12]

$$\langle u_i' u_j' \rangle = K \left[\frac{2}{3} \delta_{ij} + \frac{(1-\gamma)\left(\frac{P_{ij}}{\varepsilon} - \frac{2}{3}\delta_{ij}\frac{P}{\varepsilon}\right) + (1-c_{3p})\left(\frac{G_{ij}}{\varepsilon} - \frac{2}{3}\delta_{ij}\frac{G}{\varepsilon}\right) + \frac{S_{ij}}{\varepsilon}}{c_{1p} + \left(\frac{P+G}{\varepsilon} - 1\right)} \right], \tag{10}$$

$$\langle u_i' T' \rangle = \frac{K}{\varepsilon} \frac{\langle u_i' u_l' \rangle \frac{\partial T}{\partial x_l} + (1-c_{2T})\left(\langle u_l' T' \rangle \frac{\partial u_i}{\partial x_l} + \beta g_i \langle T'^2 \rangle\right) + S_{iT}}{c_{1T} + \frac{1}{2}\left(\frac{P+G}{\varepsilon} - 1\right)}. \tag{11}$$

The expressions for P_{ij}, G_{ij}, P, G as well as the terms S_{ij}, S_{iT} governing the influence of the wall are not presented here because of their awkwardness. Determination of these terms and the magnitudes of the constants in this model can be found in [12]. On the basis of the solution obtained by using the K–ε turbulence model, the system (10) and (11) was solved. Presented in Fig. 4 is the dependence of the dimensionless correlation $\langle u'T' \rangle_b = \langle u'T' \rangle / [U(T_W - T_\infty)]$ on the transverse coordinate. It is seen that this correlation is negative in the domain between the wall and the maximum velocity coordinate.

The results obtained in this paper indicate the necessity to take into account the singularities of free-convective flow in the construction of numerical models of turbulence, especially for the description of the flow in the domain between the wall and the maximum velocity coordinate.

NOTATION

a, thermal diffusivity coefficient, e, dimensionless rate of dissipation of the turbulent velocity fluctuation kinetic energy; $F = u/U$, dimensionless longitudinal component of the average velocity; $U = [g\beta(T_W - T_\infty)x]^{1/2}$, velocity scale; Gr, Grasshopf number; g, acceleration of gravity; K, kinetic energy of the turbulent velocity fluctuations; K_{max}, maximum value of the kinetic energy of the turbulent velocity fluctuations; k, dimensionless kinetic energy of the turbulent velocity fluctuations; Pr, Prandtl number; $q = \langle T'^2 \rangle$, average of the temperature fluctuations squared; $Re_T = K^2/(\nu\varepsilon)$, turbulent Reynolds number; $Ri_T = g\beta|\partial T/\partial y|/(\partial u/\partial y)^2$, Richardson number; T, average temperature; T_w, surface temperature, T_∞, temperature at the outer boundary of the boundary layer; u, v, velocity vector components in Cartesian coordinates; x, y, rectangular Cartesian coordinates; y_M, maximum thickness; β, coefficient of volume expansion; δ, boundary layer thickness; ε, rate of dissipation of the kinetic energy of the turbulent velocity flucatuations; ε_{max}, maximal value of the rate of kinetic energy dissipation of the turbulent velocity fluctuations; ν, ν_T, kinetic viscosity coefficient and its turbulent analog; $\tau = -\langle u'v' \rangle/(\nu/x\,Gr^{1/3})^2$, dimensionless turbulent friction; $p = \nu_T/\nu(\partial F/\partial\eta)^2$, dimensionless turbulence generation due to shear flow; $\eta = y/xGr^{1/4}$, $\eta_c = y/xGr^{1/3}$, dimensionless transverse coordinates.

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